Theory of Fine Sediment Infiltration into Immobile Gravel Bed

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Abstract: A theoretical model is developed to describe the process of fine sediment infiltration into immobile coarse sediment deposits. The governing equations are derived from mass conservation and the assumption that the amount of fine sediment deposition per unit vertical travel distance into the deposit is either constant or increases with increasing fine sediment fraction. Model results demonstrate that fine sediment accumulation decreases rapidly with depth into coarse substrate initially void of fine sediment, which is consistent with experimental observations that significant fine sediment infiltration occurs to only a shallow depth. Comparisons of the theory with flume data indicate that the model adequately reproduced the weighted-averaged fine sediment fraction values from experiments. An early model developed by Sakthivadivel and Einstein for fine sediment infiltration is in part based on the generally incorrect assumption that intragravel flow remains constant following fine sediment infiltration. Applying a correction to the Sakthivadivel and Einstein model based on alternate hypothesis that introgravel flow is driven by a constant head gives similar results as the proposed model.

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Introduction

The fraction of fine sediment in the subsurface material in a gravel-bedded river has important biological and engineering implications. Salmonids, for example, depend on relatively clean gravel deposits with appropriate hydraulic conditions for reproduction, and excessive fine sediment infiltration can harm the eggs and hatchlings in many ways (Cooper 1965; Koski 1966; Greig et al. 2005a,b, 2007). Elevated fine sediment fractions in bed material deposits decrease the porosity and hydraulic conductivity, resulting in increased energy consumption for infiltration wells that draw water from directly underneath the river bed. While fine sediment can be incorporated into bed material deposits through concurrent deposition of both fine and coarse particles, fine sediment can also infiltrate the deposit driven either by gravity or intragravel flow while the bed material is immobile.

Pioneering experiments examining fine sediment infiltration into immobile gravel beds by Einstein (1968) in a shallow laboratory flume demonstrated that fine sediment settled at the flume bottom and gradually filled the bed material pores upward. Following the work of Einstein (1968), there have been many laboratory and field studies on the subject of fine sediment infiltration (Beschta and Jackson 1979; Frostick et al. 1984; Carling 1984; Diplas and Parker 1985; Lisle 1989; Schälchli 1992). A common observation from these studies was that fine sediment infiltrated to a finite depth, usually to a depth within a few diameters of the largest bed material particles rather than settling to the bottom as Einstein (1968) originally observed.

In addition to field and laboratory experiments, Sakthivadivel and Einstein (1970) developed a theory to describe the process of fine sediment infiltration through a porous column due to intragravel flow based on the conservation of mass for fine sediment and by correlating the probability of fine sediment particles lodging in place with intragravel flow velocity. Lauck (1991) developed a model to describe the infiltration process, which assumed sediment infiltration occurs as a stochastic process of particles dropping into a predefined space. The Lauck (1991) model agreed qualitatively with observations that fine sediment infiltrates to a limited depth. However, he also demonstrated that if the thickness of the bed material was not thick enough, fine sediment filling pores from the bottom would occur [as observed by Einstein (1968)]. In his stochastically based numerical model, Lauck (1991) proposed that, once a fine sediment particle enters the pores of the bed material, it will either continue to move downward within the pores or become lodged within the bed matrix according to a quantifiable probability distribution. After a fine sediment particle is lodged in place, it becomes permanently fixed in place, which decreases the pore size opening and increases the probability for subsequent incoming fine sediment particles to become lodged. This process results in a decreased fine sediment fraction with depth into the deposit. Eventually, the pore spaces in the top layer of the bed material will be completely clogged with

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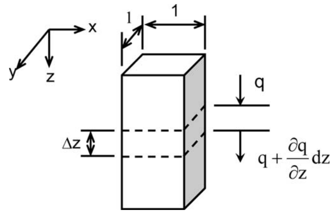

Fig. 1. Schematic diagram showing the finite volume used for the derivation of governing equations for fine sediment infiltration

fine sediment particles (i.e., the deposit becomes saturated with fine sediment) and effectively stops additional fine sediment infiltration. Herein, a coarse sediment deposit is defined as saturated with fine sediment when the pore spaces of the deposit become so small that fine sediment can no longer advance through it. The processes described above are independent of how the fine sediment particles move downward within the deposit framework (e.g., by gravity or intragravel flow) because lodging of the sediment particles is assumed solely a function of pore space geometry.

In this paper, we derive governing equations for fine sediment infiltration into immobile coarse sediment deposits based primarily on the model of Lauck (1991). Here, fine sediment is defined as any particle smaller than the initial pore size of the clean coarse sediment deposit. We introduce a trapping coefficient, which is defined as the fraction of the fine sediment load trapped in the bed material as it travels a unit vertical distance. We derive the governing equations (referred to as the model hereafter), solve them analytically for a simple case and numerically for a general case, and then compare model results with experimental results from three flume experiments reported in Wooster et al. (2008). We also apply the model to several practical scenarios, including an examination of fine sediment infiltration into a shallow deposit, fine sediment infiltration driven by intragravel flow previously defined and solved by Sakthivadivel and Einstein (1970), and infiltration of fine sediment into a stratified sediment deposit. During our reexamination of the Sakthivadivel and Einstein (1970) problem, a flaw is identified in their theory and a correction is proposed.

Governing Equations for Fine Sediment Infiltration into an Immobile Coarse Deposit

With the help of a finite volume as shown in Fig. 1, the continuity equation for fine sediment infiltrating an immobile gravel bed can be derived as

$$\frac{\partial F}{\partial t} + \frac{\partial q}{\partial z} = 0 \tag{1}$$

in which F denotes fraction of fine sediment in the deposit relative to the total volume (i.e., fine sediment volume over the sum of all solid and pore space volume, referred to as F fraction hearafter); q denotes downward fine sediment flux per unit area (solid volume per unit time per unit area); t denotes time; and z denotes depth into the sediment deposit. Both F and q are functions of space and time, i.e., $q=q(z,t)$, and $F=F(z,t)$.

As the fine sediment infiltrates downward, some particles will be trapped within the finite volume and the remaining particles will continue to move downward. We introduce β to denote a trapping coefficient, defined earlier as the fraction of the fine sediment load trapped in the bed when traveling vertically a unit distance. Because of the distance factor in the above definition, $0<\beta<\infty$, because all the fine sediment can be trapped in place for an infinitively short distance. The volume of fine sediment entering a finite volume (Fig. 1) for a small time interval Δt is $q\Delta t$, and, thus, the amount of fine sediment trapped within the finite volume during time interval Δt will be $\Delta V_f=\beta q \Delta t \Delta z =(\partial F)/(\partial t)\Delta t \Delta z$, in which Δz denotes the depth of the finite volume as shown in Fig. 1. Combining this relation with Eq. (1), we obtain

$$\frac{\partial F}{\partial t} = \beta q \tag{2a}$$

$$\frac{1}{q}\frac{\partial q}{\partial z} = -\beta \tag{2b}$$

For a given coarse sediment deposit, β will be a function of the amount of fine sediment trapped in the deposit, i.e., $\beta=\beta(F)$.

Eqs. (1) and (2a) are identical to the governing equations derived in Sakthivadivel and Einstein (1970), except for the formulations for β, which has not yet been specified in our analysis. Despite the similarities between the governing equations, the derivation presented herein is different from that of Sakthivadivel and Einstein (1970).

For a given sediment deposit, β should take a minimum value when the sediment deposit is void of fine sediment because it offers the maximum pore space for fine sediment to pass through (i.e., fine sediment passes through more easily due to the maximum pore size). As sediment accumulates in the pores (i.e., as F increases), β should also increase because the decreasing pore space makes it more difficult for fine sediment to pass through. In contrast, the trapping coefficient in the Sakthivadivel and Einstein (1970) theory decreases with an increasing fine sediment fraction in the bed, which produces results seemingly at odds with experimental observations (Beschta and Jackson 1979; Wooster et al. 2008).

Based on the hypothesis that β is either a constant or is at a minimum value when a sediment deposit is void of fine sediment and increases monotonically with increases in F, we propose the following model for β:

$$\beta = \beta_0 \exp\left(\phi \frac{F/F_s}{1-F/F_s}\right) \tag{3}$$

where β_0=trapping coefficient for the sediment deposit void of fine sediment ($F=0$); ϕ=dimensionless coefficient with a nonnegative value that defines the shape of the β to F/F_s relation; and F_s=value of F when the deposit is saturated with fine sediment. An important property inherent in Eq. (3) is that β will have a constant value (i.e., is independent of F) if ϕ is set to zero. Eq. (3) is plotted for five values of ϕ in Fig. 2, demonstrating that by varying ϕ, the function can describe a wide range of possible relations between β and F/F_s.

Substituting Eq. (3) into Eq. (2b) yields

$$\frac{1}{q}\frac{\partial q}{\partial z} = -\beta_0 \exp\left(\phi \frac{F/F_s}{1-F/F_s}\right) \tag{4}$$

Eqs. (1) and (4) form a pair of partial differential equations for $q(z,t)$ and $F(z,t)$ that can be solved numerically if coefficients β_0 and ϕ are given and boundary and initial conditions are specified. Here we examine a simple case where fine sediment infiltrates the surface of an immobile coarse sediment deposit initially void of

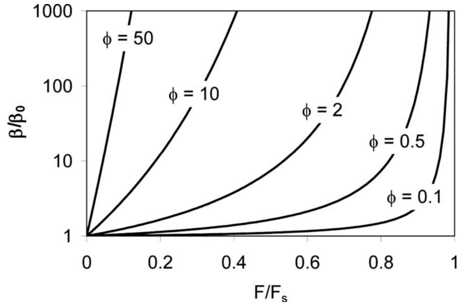

Fig. 2. Normalized trapping coefficient (β/β_0) as a function of the normalized F fraction (F/F_s) using different values of the dimensionless coefficient ϕ

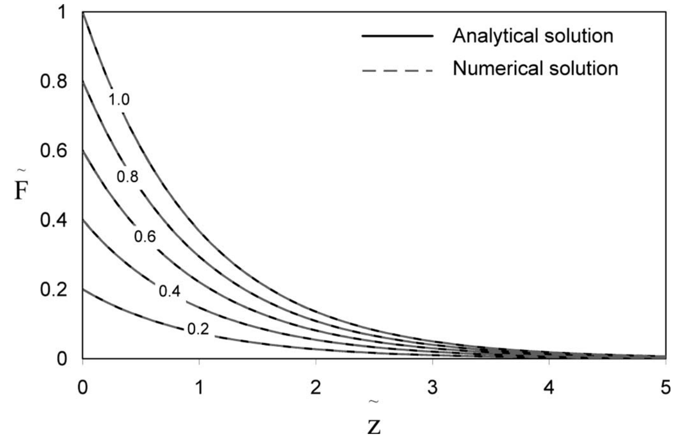

Fig. 3. $\tilde{F}$ values calculated with the analytical solution [Eqs. (9*a*) and (9*b*)] while assuming that the trapping coefficient is independent of the fine sediment fraction (i.e., $\phi=0$). Numerical solutions for $\tilde{F}$ values are also shown, which overlay almost directly on top of the analytical results. The numerical label on each curve represents $\tilde{t}$ value.

fine sediment at a constant rate prior to the surface being saturated. The initial conditions can, thus, be specified as

$$F(z,0)=0 \tag{5a}$$

$$q(z,0)=0 \tag{5b}$$

For a constant infiltration rate at the surface of the deposit prior to the surface being saturated with fine sediment, the boundary condition at the surface of the deposit is specified as

$$q(0,t)=\begin{cases} q_0 & \text{for } F(0,t)<F_s \\ 0 & \text{for } F(0,t)=F_s \end{cases} \tag{6}$$

where q_0 denotes the constant fine sediment infiltration rate at the surface of the coarse sediment deposit. No downwind boundary condition is needed for the case of fine sediment infiltrating an infinitely deep gravel deposit, or $F=0$ at $z\to\infty$.

In the following, it is found convenient to discuss results in terms of dimensionless variables, $\tilde{F}=F/F_s$, $\tilde{q}=q/q_0$, $\tilde{z}=\beta_0 z$, and $\tilde{t}=(t)/[F_s/(\beta_0 q_0)]$, which translate Eqs. (1), (4), (5*a*), (5*b*), and (6) to:

$$\frac{\partial\tilde{F}}{\partial\tilde{t}}+\frac{\partial\tilde{q}}{\partial\tilde{z}}=0 \tag{7a}$$

$$\frac{1}{\tilde{q}}\frac{\partial\tilde{q}}{\partial\tilde{z}}=-\exp\left(\phi\frac{\tilde{F}}{1-\tilde{F}}\right) \tag{7b}$$

$$\tilde{F}(\tilde{z},0)=0 \tag{8a}$$

$$\tilde{q}(\tilde{z},0)=0 \tag{8b}$$

$$\tilde{q}(0,\tilde{t})=\begin{cases} 1 & \text{for } \tilde{F}(0,\tilde{t})<1 \\ 0 & \text{for } \tilde{F}(0,t)=1 \end{cases} \tag{8c}$$

The normalized equations above eliminated β_0 and q_0 and retained only one parameter [coefficient ϕ in Eq. (7*b*)]. An analytical solution for the simplified case where β is assumed to be independent of F (i.e., $\beta=\beta_0$, or $\phi=0$) is obtained below in Eqs. (9*a*) and (9*b*) and presented in Fig. 3

$$\tilde{F}=\tilde{F}(0,\tilde{t})\exp(-\tilde{z}) \tag{9a}$$

$$\tilde{F}(0,\tilde{t})=\begin{cases} \tilde{t} & \text{for } \tilde{t}<1 \\ 1 & \text{for } \tilde{t}\geqslant 1 \end{cases} \tag{9b}$$

General Numerical Solution to the Nonsimplified Equation

For conditions where β is not constant (or $\phi\neq 0$), an analytical solution has not been found, and Eqs. (7*a*) and (7*b*) subject to initial and boundary conditions in Eqs. (8*a*)–(8*c*) were solved numerically. Numerical solution results for $\phi=0$, presented in Fig. 3, agree with the available analytical solution. Results for $\phi=0.1$, 0.5, 1, and 10 are presented in Fig. 4, and the equilibrium profiles for these four runs are compared in Fig. 5. Comparing the $\tilde{F}$ profiles under different ϕ values shown in Figs. 4 and 5 demonstrates that $\tilde{F}$ to $\tilde{z}$ profiles become progressively steeper with an increase in ϕ. In addition, the time needed for infiltration to reach an equilibrium decreases with an increase in ϕ (Fig. 4). The increased β results in a steeper slope in the $\tilde{F}$ profiles and causes the surface to reach saturation (i.e., $\tilde{F}=1$) more rapidly, which implies the realization of an equilibrium state because no additional fine sediment can pass through the surface layer to infiltrate further downward.

Examination of the Theory with Flume Experimental Data

Three flume experimental runs were conducted in order to examine the effects of varying the grain-size distribution of the bed material on fine sediment infiltration into an immobile gravel bed. Details of the experiments, their results, and data analysis are reported in Wooster et al. (2008); only a brief overview of the experiments and the relevant results are provided.

The experiments were conducted in a 28 m long, 0.86 m wide, and 0.9 m deep flume located at the Richmond Field Station (RFS), University of California. Nine gravel mixtures with different grain size distributions and void of fine sediment (<2 mm), were placed in 10 zones at a thickness of 16 cm to serve as the initial bed material for each of the three runs. The geometric mean sizes of the bed material in the 10 zones ranged from 4.2 to 17.2 mm, and their geometric standard deviations ranged

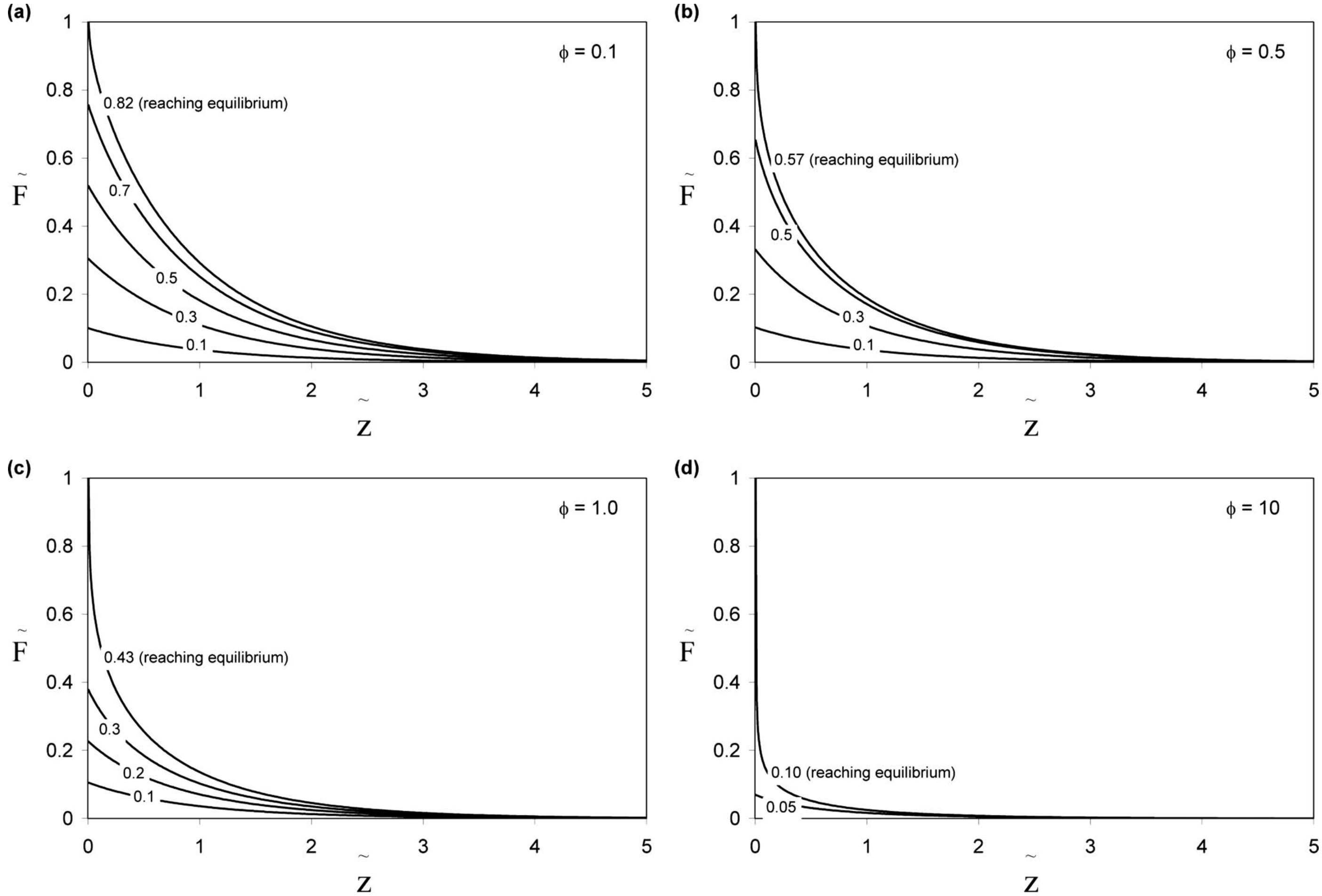

Fig. 4. Simulated evolution of $\widetilde{F}$ under four different ϕ values: (a) $\phi=0.1$; (b) $\phi=0.5$; (c) $\phi=1.0$; and (d) $\phi=10$. The numerical label on each curve represents $\widetilde{t}$ value.

from 1.17 to 1.90. A constant discharge of 40 l/s was used throughout all runs and the bed slope was set at 0.004. The experimental conditions resulted in minimum bed material mobilization during the experiments. The fine sediment used for infiltration had a geometric mean grain size of 0.35 mm and a geometric standard deviation of 1.24. Fine sediment was introduced from the upstream end of the flume at 0.209 kg/min., 2.09 kg/min, and 20.9 kg/min for 100 hr, 10 hr, and 1 hr, respectively. Bed material beneath a surface layer of approximately two bed material median grain size diameters was sampled in vertical layers at six locations in each zone following each run. In addition, Wooster et al. (2008) also mixed 35 unimodal sediment samples with grain sizes ranging between 0.075 and 22 mm to test their porosity and obtain a relation between porosity and geometric standard deviation for a unimodal sediment deposit. Combining the fine sediment fraction and the grain size distribution data from the infiltration experiments with the porosity to geometric standard deviation relation, Wooster et al. (2008) obtained the following relations (notation modified to be consistent with this paper):

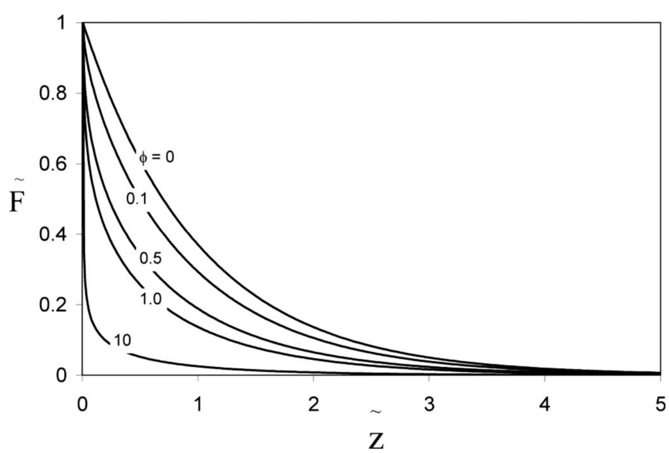

Fig. 5. Simulated equilibrium $\widetilde{F}$ profile under five different ϕ values: 0, 0.1, 0.5, 1.0, and 10. The numerical label on each curve represents ϕ value.

$$\frac{f}{f_s} = \exp(-\beta z) \tag{10a}$$

or

$$\widetilde{f} = \exp(-\widetilde{z}) \tag{10b}$$

$$f_s = \frac{0.621(1-0.621\sigma_{sg}^{-0.659})\sigma_{gg}^{-0.659}}{1-0.621^2(\sigma_{sg}\sigma_{gg})^{-0.659}} \times \left[1-\exp\left(-0.0146\frac{D_{gg}}{D_{sg}}+0.0117\right)\right] \tag{11}$$

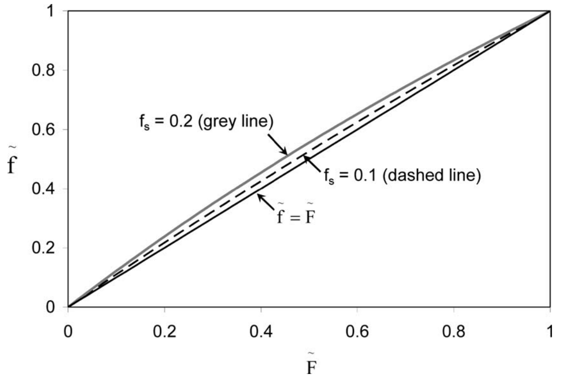

Fig. 6. Comparison between $\tilde{f}$ and $\tilde{F}$ for $f_s=0.1$ and $f_s=0.2$, indicating the two are close to each other and can be used interchangeably in almost all the practical problems

$$\beta = \frac{0.0233\sigma_{gg}^{1.95}}{D_{gg}}\left[\ln\left(\frac{D_{gg}\sigma_{gg}}{D_{sg}}\right) - 2.44\right] \qquad (12)$$

in which f=the fraction of fine sediment in the deposit relative to the total solid volume (i.e., fine sediment volume over the sum of fine and coarse sediment volume and excluding pore space volume); f_s=the value of f when the deposit is saturated with fine sediment; $\tilde{f}=f/f_s$; σ_{gg} denotes bed material geometric standard deviation; σ_{sg} denotes infiltrating fine sediment geometric standard deviation; D_{gg} denotes bed material geometric mean grain size; D_{sg} denotes infiltrating fine sediment geometric mean grain size; and z denotes the depth into the bed material measured from the bottom of the surface layer (assumed to be $2D_{gg}$ thick). Eq. (10*b*) is identical to the analytical solution under equilibrium conditions presented in this paper [Eq. (9*a*) for $\tilde{t} \geqslant 1$] except that $\tilde{F}$ in Eq. (9*a*) is replaced with $\tilde{f}$ in Eq. (10*b*). The two variables $\tilde{f}$ and $\tilde{F}$ are related by

$$f = \frac{F}{1-\lambda_g + F} \qquad (13a)$$

$$F = (1-\lambda_g)\frac{f}{1-f} \qquad (13b)$$

in which λ_g denotes the porosity of the clean gravel deposit (i.e., porosity when $F=0$).

Data from Wooster et al. (2008) show that, in general, $0.1 < f_s < 0.2$. A comparison of $\tilde{f}$ and $\tilde{F}$ for $f_s=0.1$ and $f_s=0.2$ is presented in Fig. 6, indicating that $\tilde{f}$ and $\tilde{F}$ are very close to each other and can be used interchangeably for almost all the practical problems. Because of that, we can consider Eq. (10*b*) as a close approximation of Eq. (9*a*) under equilibrium conditions (i.e., $\tilde{t} \geqslant 1$) and compare experimental data for $\tilde{f}$ values with Eq. (10*b*) to validate Eq. (9*a*).

Eq. (10*b*) is compared with measured experimental data and a weighted average of the experimental data in Fig. 7. Despite the large variance in the experimental data due to the stochastic nature of the physical process of fine sediment infiltration, results indicate that Eq. (10*b*) closely approximates the weighted-averaged experimental data. Wooster et al. (2008) calculated the root-mean-square error (RMSE) of Eq. (10*b*) as 0.073 relative to the weighted-averaged experimental data. The good agreement between the weighted-averaged experimental data and model predictions suggests the following for fine sediment infiltrating into an immobile bed initially void of fine sediment: (a) Eqs. (10*a*), (10*b*), (11), and (12) adequately represent the equilibrium fine sediment fraction in the bed material; and (b) assuming a constant β (i.e., independent of fine sediment accumulation) is consistent with experimental observations.

Examination of Fine Sediment Infiltration into a Deposit of Limited Depth

Einstein (1968) observed during his flume experiments that fine sediment first settles at the bottom of the flume and gradually fills the pores of the sediment deposit upward. Subsequent experimental results (Beschta and Jackson 1979; Frostick et al. 1984; Carling 1984; Diplas and Parker 1985; Lisle 1989; Schälchli 1992) and the findings reported here, however, contradict this observation. Lauck (1991) suggested that the Einstein (1968) observations were an artifact of the thin deposits used in his experiments. Here we test this potential explanation with our model by specifying an additional lower boundary condition to the governing equations [Eqs. (9), (10*a*), and (10*b*)] so that no fine sediment can pass through the flume bottom

$$\tilde{q}(\tilde{z}_{max}, \tilde{t}) = 0 \qquad (14)$$

where $\tilde{z}_{max}$ denotes the bottom of a flume. Results for F from numerical simulations with $\phi=0$ for two $\tilde{z}_{max}$ values: (1) $\tilde{z}_{max}$

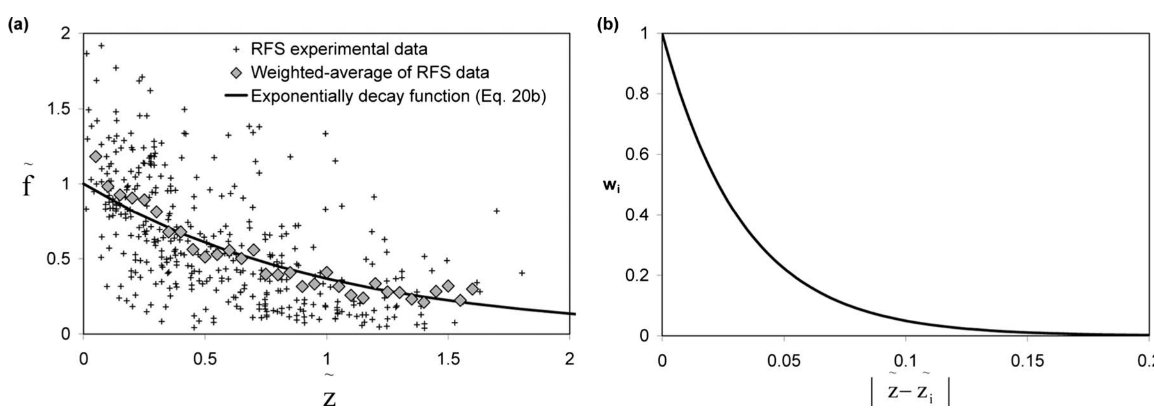

Fig. 7. (a) Comparison of the RFS experimental data with Eq. (10*b*). The weighted-averaged values are calculated as $\tilde{f}|_{\text{weighted average}} = \Sigma(w_i\tilde{f})/\Sigma w_i$ with a weighting factor $w_i=\exp[-30|\tilde{z}-\tilde{z}_i|]$, as shown in (b) [see Wooster et al. (2008) for details].

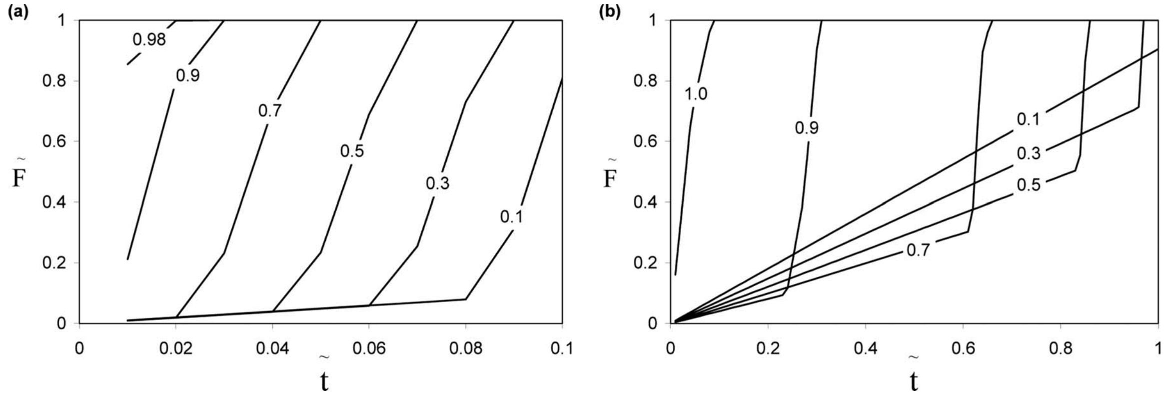

Fig. 8. Simulated evolution of $\tilde{F}$ by setting an impermeable lower boundary at $z=\tilde{z}_{max}$ that no fine sediment can infiltrate through, ϕ was set to zero for both runs, (a) $\tilde{z}_{max}=0.1$; and (b) $\tilde{z}_{max}=1.0$. The numerical label on each curve represents $\tilde{z}/\tilde{z}_{max}$ value (i.e., 0 represents the surface and 1 represents the bottom of the deposit). Diagrams show bottom-first and subsequent upward filling patterns as evidenced by observing the $\tilde{t}$ values of different curves at $\tilde{F}=1$ (i.e., deposit is saturated with fine sediment), diagram (a), for example, shows that the deposit near the surface ($\tilde{z}/\tilde{z}_{max}=0.1$) will not be saturated with fine sediment until $\tilde{t}>0.1$, while the deposit near the bottom ($\tilde{z}/\tilde{z}_{max}=0.98$) will be saturated with fine sediment at approximately $\tilde{t}=0.02$.

$=0.1$ and (2) $\tilde{z}_{max}=1.0$ are presented in Fig. 8. In both cases, a bottom-first and subsequent upward filling pattern is predicted, similar to what Einstein (1968) observed in his flume experiments. This supports the Lauck (1991) hypothesis that the limited sediment deposit depth in the Einstein (1968) experiments was most likely the cause for observing the bottom-first and upward filling trend.

An upward filling of the deposit with fine sediment can also occur if no fine sediment is trapped in the pores as it travels downward (i.e., $\beta=0$). This scenario, although possible, is very unlikely because some fine sediment should lodge in place where coarse particles are in contact with one another. The solutions to the governing equations presented in Fig. 6 imply that any fine sediment trapping in the deposit will result in the eventual saturation of the surface layer with fine sediment and a decreased fine sediment fraction at depth, assuming there is no limitations in the depth of the coarse deposit.

Reexamination of the Sakthivadivel–Einstein (1970) Theory

Sakthivadivel and Einstein (1970) developed a theory describing the process of clogging a porous sediment column by fine sediment carried by intragravel flow. Assuming that particles can be trapped at a given time at a specific location, they derived governing equations to describe the process of fine sediment infiltration. These equations are identical to Eqs. (1) and (2*a*) but their model for β was different. Based on their experimental observations that the probability for a fine sediment particle becoming trapped (which is proportional to β) is inversely proportional to intragravel flow velocity, and assuming that discharge through the porous column remains constant, Sakthivadivel and Einstein (1970) proposed the following formulation for β:

$$\beta = \frac{\lambda_p}{\lambda_g}\beta_0 = \left(1 - \frac{F}{\lambda_g}\right)\beta_0 \tag{15}$$

We have introduced a minor correction in Eq. (15) from the original presentation of Sakthivadivel and Einstein (1970) by replacing $(1+\lambda_s)F$ with F, in which λ_s denotes the porosity of the infiltrating fine sediment following its deposition. This minor correction does not affect the general behavior of the governing equations, although the base parameters used for parameter normalization are slightly different. Normalizing the variables with appropriate base-parameters, Sakthivadivel and Einstein (1970) obtained the following equation:

$$\frac{1}{\tilde{q}}\frac{\partial \tilde{q}}{\partial \tilde{z}} = -(1-\tilde{F}) \tag{16}$$

The normalizations of the parameters in Eq. (16) are not necessarily identical to the normalizations provided in this paper. However, the differences do not prevent a direct comparison of the general behavior between Eqs. (7*b*) and (16), and, thus, the same set of notations are employed here for simplicity. The solution to Eq. (16), coupled with the continuity equation [Eq. (7*a*)] and the initial and boundary conditions specified in Eqs. (8*a*)–(8*c*), was provided by Sakthivadivel and Einstein (1970) as

$$\tilde{F} = \left[1 + \frac{\exp(\tilde{z})}{\exp(\tilde{t}) - 1}\right]^{-1} \tag{17}$$

Eq. (17) dictates that $\tilde{F}\rightarrow 1$ everywhere for finite $\tilde{z}$ as $\tilde{t}\rightarrow\infty$, i.e., the deposit will eventually be filled with fine sediment as shown in Fig. 9. The speed at which the fine sediment front advances ($d\tilde{z}/d\tilde{t}$) implied by Eq. (17) is found to be

$$\frac{d\tilde{z}}{d\tilde{t}} = \frac{1}{1 - e^{-\tilde{t}}} \tag{18}$$

Eq. (18) implies that the fine sediment front will advance downward at a constant speed once the infiltration time becomes sufficiently long (i.e., $d\tilde{z}/d\tilde{t}\rightarrow 1$ as $\tilde{t}\rightarrow\infty$), clogging the entire coarse deposit. Under natural conditions, however, fine sediment usually clogs only a thin layer near the surface of the deposit, which prevents further infiltration and clogging of the sediment deposit at deeper depths (Frostick et al. 1984; Lisle 1989). The flaw in the Sakthivadivel and Einstein theory is attributed to their model for β. As shown in Eq. (15), their β decreases with an increase in F, i.e., the more fine sediment stored in the pores, the less efficient the trapping, which does not seem plausible. This prediction is likely an artifact of their assumption that downward transport is

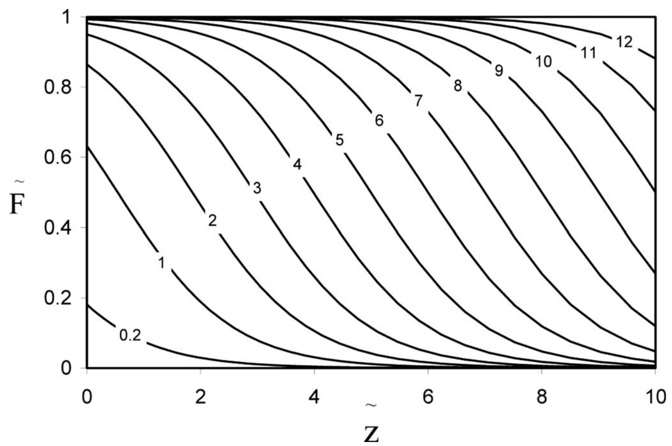

Fig. 9. The evolution of fine sediment fraction in a gravel deposit due to intragravel flow, based on the original theory of Sakthivadivel and Einstein (1970) without our proposed modifications, showing that the deposit will eventually fill with fine sediment, i.e., $\tilde{F}(\tilde{z},\tilde{t})_{\tilde{t}\to\infty}=1$. The numerical label on each curve represents $\tilde{t}$ value.

driven by a constant water discharge. This would require that as sediment accumulates and the porosity decreases, the intragravel velocity must increase, which results in a decrease in the predicted trapping rate. In the experiments conducted by Sakthivadivel and Einstein (1970), a constant water discharge through the porous column may be a reasonable approximation, but this is likely unrealistic under natural conditions. More realistically, a sediment deposit experiences a constant hydraulic head that drives the intragravel flow. Once the pores of the sediment deposit are clogged with fine sediment, the hydraulic conductivity of the sediment deposit will decrease, resulting in a decreased discharge through the porous column. As a result, intragravel velocity decreases with an increase in the fine sediment fraction. An alternate analysis based exclusively on information provided by Sakthivadivel and Einstein (1970), with the modified more realistic assumption of a constant hydraulic head and the minor correction introduced to Eq. (15) earlier is presented below. Two equations directly adapted from Sakthivadivel and Einstein (1970) are

$$\beta \propto \frac{1}{u_i} \tag{19a}$$

$$K \propto \frac{\lambda_p^3}{(1-\lambda_p)^2} \tag{19b}$$

where u_i denotes intragravel flow velocity, and K denotes hydraulic conductivity. Using q_w to denote water discharge per unit area, and assuming hydraulic head remains constant, we obtain the following two relations:

$$u_i = \frac{q_w}{\lambda_p} \tag{20a}$$

$$q_w \propto K|_{\text{averaged over the entire depth}} \tag{20b}$$

Manipulating Eqs. (19*a*), (19*b*), (20*a*), and (20*b*), we obtain

$$\beta \propto \frac{1}{H}\int_0^H (1/\lambda_p^2 - 1)dz \tag{21}$$

where H denotes the thickness of the sediment deposit where hydraulic head remains constant. Because the initial porosity (λ_g) applies through the deposit when the coarse sediment is void of fine sediment (and $\beta=\beta_0$), the following model for β is obtained:

$$\beta = \frac{\int_0^H (1/\lambda_p^2 - 1)dz}{H(1/\lambda_g^2 - 1)}\beta_0 \tag{22}$$

Because λ_p decreases with an increase in F, β increases as F increases based on Eq. (22). This dependency mimics that of Eq. (3). Because β increases with an increase in F, the process of fine sediment infiltration due to intragravel flow is similar to the proposed model.

Infiltration of Fine Sediment into a Multiple-Layer Filter

Multiple-layer filters are often used in hydraulic projects such as at the toe of an earth filled dam. Here we simulate the infiltration of fine sediment into a multiple-layer filter to demonstrate a potential application of our theory. Infiltration of fine sediment into multiple layers of a coarse sediment deposit cannot be simulated with the normalized equations because β_0 values for different layers are different. Alternatively, we developed a numerical solution based on the nonnormalized governing equations and boundary conditions [Eqs. (1), (4), (5*a*), (5*b*), and (6)], in combination with the semiempirical relations developed in Wooster et al. (2008) [Eqs. (10*a*), (11), and (12) herein]. Simulations are conducted for two runs, each with five 5 cm thick layers of different sized gravel: Run 1's top layer has a geometric mean size of 32 mm and each subsequent underlying layer is finer by a half-phi scale (i.e., geometric mean grain size decreases by a factor of $\sqrt{2}$); and Run 2's top layer has a geometric mean size of 8 mm and each subsequent underlying layer is coarser by a half-phi scale (i.e., geometric mean grain size increases by a factor of $\sqrt{2}$). The geometric standard deviation for each of the sediment layers for both runs is assumed to be 2.0. Fine sediment that infiltrates the deposit is assumed to have a geometric mean grain size of 0.1 mm and a geometric standard deviation of 1.3. In addition to the two multiple layer runs, for comparative purposes, two additional runs were conducted for sediment deposits without stratification: Run 1u with a deposit identical to that at the top layer of Run 1, and Run 2u with a deposit identical to that at the top layer of Run 2. A summary of the parameters for the four runs described above and the calculated β values are presented in Table 1. Results of the four runs are presented in Fig. 10, illustrating the potential application of the model in hydraulic projects using multiple-layer filters or other applications involving physically based filters (i.e., the filtering mechanism is not based on chemical or biological processes). As observed in Fig. 10, the simulated fine sediment fraction spikes up at the interface between different subsurface layers where the lower layer is finer than the layer above (Run 1). This spike in fine sediment fraction is caused by the finer grain size in the lower layer being more difficult for fine sediment to infiltrate through (higher β value), which results in a faster accumulation of fine sediment due to the higher input rate (infiltration) from the upper layer. Conversely, simulated fine sediment fraction spikes down at the interface between layers where the lower layer is coarser than the layer above (Run 2), which is caused by the coarser lower layer being easier for fine sediment to infiltrate through (lower β value) and subsequently less fine sediment retention compared to if the deposit is homogeneous (Run 2u).

Table 1. Parameters for Simulations of Multiple Subsurface Sediment Layers and the Calculated Trapping Coefficient β

	0–5 cm		5–10 cm		10–15 cm		15–20 cm		20–25 cm	
	D_{gg} (mm)	β (mm^{-1})	D_{gg} (mm)	β (mm^{-1})	D_{gg} (mm)	β (mm^{-1})	D_{gg} (mm)	β (mm^{-1})	D_{gg} (mm)	β (mm^{-1})
Run 1	32.0	0.0113	22.6	0.0146	16.0	0.0187	11.3	0.0237	8.0	0.0297
Run 2	8.0	0.0297	11.3	0.0237	16.0	0.0187	22.6	0.0146	32.0	0.0113
Run 1u	32.0	0.0113	32.0	0.0113	32.0	0.0113	32.0	0.0113	32.0	0.0113
Run 2u	8.0	0.0297	8.0	0.0297	8.0	0.0297	8.0	0.0297	8.0	0.0297

Discussion

Potential practical applications of our findings include assessing fine sediment impacts to groundwater pumps and impacts associated with dam removal. The threat of fine sediment clogging the bed material where groundwater wells are pumped may be minimal because fines will primarily deposit in the near surface layer. For dam removal cases that could potentially release large quantities of fine sediment to downstream reaches, our theory implies that a quicker release will be more beneficial in terms of reducing the impact from potential fine sediment infiltration. Because fine sediment infiltration only occurs to a limited depth and will cease once the surface layer is saturated, as soon as the elevated fine sediment supply recedes, a high flow that mobilizes the bed should be able to flush the bed material of any excessive fine sediment buildup.

Conclusion

We present a model to describe the process of fine sediment infiltration into immobile coarse sediment deposits. The trapping coefficient, defined as a fraction of fine sediment load trapped in the deposit for traveling a unit distance, can either be independent of the amount of fine sediment accumulated in the sediment deposit or increases as the volume of fine sediment accumulates in the bed material pores. Using a constant fine sediment trapping coefficient assumption, the vertical fine sediment fraction profile decreases exponentially with depth into the deposit. If the trapping coefficient increases as fines progressively clog pore spaces, the rate of decrease in fine sediment accumulation with depth into the deposit increases. Reexamining the theory of Sakthivadivel and Einstein (1970) for fine sediment infiltration due to intragravel flow reveals that their initial assumption that intragravel discharge remains constant following fine sediment infiltration results in an unrealistic solution. Replacing the constant intragravel discharge assumption with a more realistic assumption that hydraulic head remains constant results in a revised model that behaves similarly to that proposed in this paper. This implies that fine sediment infiltration as a result of intragravel flow is similar to fine sediment infiltration driven by gravity. The result of our proposed model that fine sediment fraction as a result of fine sediment infiltrating a clean gravel deposit decreases rapidly in depth agrees with previous field (Frostick et al. 1984; Lisle 1989) and experimental observations (Beschta and Jackson 1979; Wooster et al. 2008). This result implies that the interaction between infiltrating fine sediment and the bed material is limited to near surface sediment as long as the deposit remains immobile.

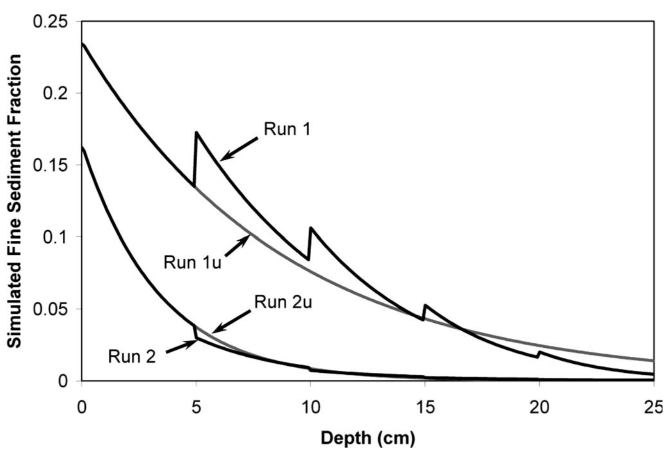

Fig. 10. Simulated fine sediment fractions as a result of fine sediment infiltrating sediment deposits with multiple subsurface layers (Runs 1 and 2). Simulated fine sediment fractions for sediment deposits without stratification (Runs 1u and 2u) are also presented for comparison purposes. Grain size distributions for Runs 1u and 2u are identical to that of the top subsurface layer for Runs 1 and 2, respectively.

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Notation

The following symbols are used in this paper:

D_{gg} = geometric mean grain size of the gravel deposit that will receive fine sediment infiltration;

D_{sg} = geometric mean grain size of the infiltrating fine sediment;

F = fine sediment fraction expressed as fine sediment volume over the sum of pores and solid volume and referred to as F fraction;

F_e = F fraction after infiltration reaches equilibrium;

F_s = F fraction when the deposit is saturated with fine sediment;

f = fine sediment fraction expressed as fine sediment volume over the sum of fine and coarse sediment volume and referred to as f fraction;

f_s = f fraction when the deposit is saturated with fine sediment;

K = hydraulic conductivity;

q = sediment infiltration rate expressed as volume per unit area per unit time;

q_0 = sediment infiltration rate expressed as volume per unit area per unit time at the surface of the deposit;

t = time;

u_i = intragravel flow velocity;

z = downward coordinate, which is equivalent to depth into the deposit;

$z_{\max}$ = depth of the sediment deposit;

β = fine sediment trapping coefficient as it infiltrates downward;

β_0 = fine sediment trapping coefficient as it infiltrates downward when $F=0$;

σ_{gg} = geometric standard deviation of the gravel deposit that will receive fine sediment infiltration;

σ_{sg} = geometric standard deviation of the infiltrating fine sediment;

ϕ = coefficient in fine sediment trapping coefficient function; and

$\tilde{}$ = variables with $\tilde{}$ are normalized with a base value.

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